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Step-Change and Drift in the Parameters of Logistic Regression Profiles Using Robust Estimators: Examination and Analysis

Abstract

It has been established in the literature that the most sought-after usual method for estimating the parameters of linear profiles is the least squares method, while the maximum likelihood estimation (MLE) is used for most non-linear profiles. Various robust-based methods have been presented to estimate the parameters of the model. The authors propose a novel method for estimating the parameters of simple linear profiles and three methods for logistic profiles. As such, Phase II profile monitoring was considered for regression profiles with contaminated data. That is, both basic assumptions considered in most studies and researches in the field of monitoring profiles are simultaneously violated here. The methods examined for logistic regression profiles are the maximum weighted likelihood estimation (MWLE) method, the ridge M-estimator method, and the weighted ridge M-estimator method, which is a combination of the first two methods. The results indicate that in the presence of outlier data, the type I error rate is highly inflated. Furthermore, the proposed calculation methods and their results were tested on step changes and drifts. Also, a test power diagram was plotted for each of the proposed methods. The findings indicated that the robust-based methods significantly outperformed the classical methods in terms of step-change and drift. Among the proposed robust methods, the weighted ridge M-estimate combined method exhibited the best performance, while the MLE method performed the worst among the four proposed methods.

Keywords: *Robust method, outlier data, logistics profiles, profile monitoring, Phase II*

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Introduction

The monitoring of linear profiles is one of the relatively new applications in statistical quality control, which has garnered great academic interest in recent years. Therein, examining binary variables is undoubtedly one of the more pronounced applications of profile monitoring (Shadman, 2017). Profile monitoring is often achieved in two phases. In Phase, I, a set of primary data that has been accumulated over time is analyzed to determine the variability of the sustainability process and its stability (Shadman, 2017), while phase II seeks to unveil as soon as possible step-changes and drifts that are allowed in the parameters. The performance of the control charts, which are paramount to statistical processes, is assessed by the average run length (ARL) evaluation index. Control charts are an important and powerful statistical process control tool. Control charts are used to control the magnitude of changes in a quality characteristic (Shadman, 2017).

Studies of the previous decades have examined control charts with qualitative characteristics that follow a known distribution. That is, statistical parameters such as mean and standard variation are the have been the foci of control charts. One of the rather significant issues in controlling any process, profiles being no exception, is determining the real-time process, as it can help pinpoint the reason the process is out of control, hence saving a great deal of time and financial resources.

As such, employing the right method for estimating the change point can't be ever understated. Thus far, common methods such as MLE and torque-based methods, among others, have been used extensively for estimation to great results, albeit there is no outlier data. Yet, in the presence of contaminated data in the dataset, the classical methods are established to not exhibit the desired efficiency. Robust methods are gaining more recognition as one of the methods with a low error rate and high accuracy in terms of change point estimation, as they offer a more appropriate estimation of the parameters by ignoring or down-weighing the outlier data.

Hakimi et al. (2018) examined the so-called robust approaches for logistic regression profile monitoring. Rakstal (2016) addressed in his studies the compatibility of parametric models using the M estimator. Pignatiol and Samuel (2015) proposed an estimator to determine the time of a step-type change in inconsistent processes, where the proposed estimator is an MLE-based method in binomial processes. Mirbeik et al. (2016) sought to propose a solution for estimating the change point for monitoring autocorrelated simple linear profiles using the MLE and clustering method. Dekoan et al. (2015) examined Phase II monitoring of generalized linear profiles using the weighted likelihood ratio test (WLRT). Shadman et al. (2017) examined the change point approach to monitor the Generalized Linear Profiles in phase II SPC. Shadman et al. (2015) also examined a change point method for monitoring Generalized Linear Profiles in Phase I. Sharifi et al. (2012) proposed a maximum likelihood estimator to identify the real-time step change in phase II monitoring of binary profiles.

Hakimi et al. (2018) proposed a robust method for Phase I monitoring of binary profiles and employed a robust-based weighted maximum likelihood method for estimating the parameters of binary profiles in Phase 1, with the ultimate purpose of mitigating the effect of outlier data on the statistical performance of T^2 control chart to reduce the probability of type 1 error for monitoring logistic profiles. They concluded that the proposed method performs better than the maximum likelihood method based on the power of the T^2 control chart.

Noorossana and Amiri (2007) examined Phase II Monitoring of linear profiles, where they employed methods such as simulation and calculation of the average run length to improve the monitoring of linear profiles in phase II. Amiri et al. (2011) examined improving statistical process monitoring of quality characteristics with the polynomial profile in phase II, where they proposed a new method using the orthogonal polynomial method, a new approach has been proposed based on orthogonal polynomial approach, in which only two control charts are used to monitor a kth-order polynomial profile. Simulation findings and ARL curve analysis imply the better performance of the proposed approach compared to the existing approach. Also, using the proposed approach is much easier in practice. Taghipour et al. (2014) examined Phase I monitoring of polynomial profiles multivariate and multivariate linear profiles using multivariate linear regression. In the first phase, the parameters of the process were uncertain, and the goal was hence to evaluate the stability of the process and estimate the parameters of the process under control. Four methods were used for the polynomial and multivariate profiles, and the efficiency of the methods was compared using simulation and power comparison of control charts in identifying change points. Also, the performance of the methods was examined in a calibration case study at NASA.

Soleimaniyan et al. (2013) examined Phase II monitoring of binary response profiles, where four control schemes, namely, Hotelling, MEWMA, likelihood ratio test (LRT), and LRT/EWMA are proposed to monitor binary response profiles in phase II. The performance of the proposed control charts is evaluated and compared by simulation experiments for different shift values in the parameters of the profile in terms of the average run length (ARL) criterion. The results show that all methods work well in the sense that they can effectively detect shifts in the process parameters. Based on the results, MEWMA and LRT/EWMA methods display a better performance for small to moderate and large shift values, respectively.

Thus far, linear profiles have been the focus of the greatest share of the profile monitoring literature. In the works that have been done in the field of zero and one profiles, changes in one point have been discussed, and the approach employed for estimating the change point is maximum likelihood estimation (MLE). In this research, the study of the effect of the shift in the values of logistics profiles has been done using stable methods, which is more accurate and comprehensive than the methods used in the past.

Research Methodology

This research employs, for its purposes, a case study containing outlier data within the observations to monitor the logistics profile. Contaminated data are of several types. Here, the parameters of the problem are estimated by assuming the contamination of some response values in a small number of profiles, following which the estimation of parameters using the conventional methods of monitoring profiles is assessed. First, the robust-based estimation methods for simple linear profiles are outlined and described. They function by weighing down the contaminated data.

In this research, three methods, namely, the weighted maximum likelihood estimation, the weighted ridge M-estimator method for estimating the parameters of logistic profiles, and the ridge M-estimator method for estimating the parameters of the linear profile in Phase II are proposed. After that, the M-estimator and simple LTS methods are described. The performance of these methods is hence measured through type I and type II error of control charts while applying the shifts.

1. Weighted maximum likelihood estimation

Carroll and Pederson (1993) proposed assigning a down-weighted coefficient to outliers in the Maximum Likelihood Estimation (MLE) method for estimating the parameters. The corresponding formula highly resembles that of the Mahalanobis distance. Maronna et al. (2006) calculated this distance for variable x as follows.

$$h_n(x) = \left((x-\mu_n)\right)\sum_{n}^{-1}\left((x-\mu_n)^{\frac{1}{2}}\right) \qquad (1)$$

Where μ_i is the vector representing the mean of y, $\sum n$ is the variance-covariance matrix of these values, and n denotes the number of observations. Since x is considered constant in the profiles and observations y are maintained on their mean, the statistical distance is calculated from the following equation:

$$h_n(y_i) = \frac{y_i - \mu_{y_i}}{\sigma_{y_i}} p_y \qquad (2)$$

Where $\mu_{y_i} = m.\pi_i$ and $\sigma_{y_i} = \sqrt{m.\pi_i(1-\pi_i)}$. In other words, the purpose of this distance is to offer a standardized measure of the distance of the observations from their average. Another name for this distance is Pearson residual. Next, the polynomial distribution function is used to offer a robust estimation method. The resulting polynomial distribution function is determined as follows:

$$P_y = \binom{m}{y}.p^y(\beta).(1-p(\beta))^{m-y} \qquad (3)$$

As such, the likelihood function for β can be rewritten as follows:

$$L(\beta) = \prod_{i=n}^{n} \binom{m}{y_i} p_i^{y_i}(\beta).\left(1 - p_i(\beta)\right)^{(m-y_i)} \quad (4)$$

By applying the logarithm function to both sides of the equation and leave out the data that do not depend on the β parameter:

$$log\, L(\beta) = \sum_{i=1}^{n} \{y_i\, log\, p_i\,(\beta) + (m - y_i)\, log\, 1 - p_i(\beta))\} \quad (5)$$

Minimizing the above function would yield a robust estimate for obtaining the parameters. The following equation also takes into consideration the intention to down-weight outliers:

$$\sum_{i=1}^{n} w_i(y_i\, log\, p_i\,(\beta) + (m - y_i)(log(1 - p_i(\beta))) \quad (6)$$

In terms of w_i :

$$w_i = w(h_n(y_i) \quad (7)$$

Where w is a non-increasing function, Carroll and Pederson (1993) provided the following function for calculating w (c < 0).

$$W(u) = \left(1 - \frac{\mu^2}{c^2}\right)^3 |\mu| \le c \quad (8)$$

In case $|\mu| \le c$, the value of function I will be equal to 1, otherwise zero.

2. Ridge M-estimates

Maronna et al. (2006) also employed a method M-estimator-based method to estimate the regression parameters. For the MLE method, the purpose is to minimize the following equation:

$$M(\beta) = \sum_{i=1}^{n} d^2\left(\mu_{y_i} - y_i\right) \quad (9)$$

Where $\mu_{y_i} = m.\pi_i$. that is, for $\mu_{y_i} = \mu$, $y_i = s$, the distance $d(\mu, s)$ will be calculated as follows:

$$d(u, s) = \{2(log\, u + (m - s)\, log(m - u)\}^{\frac{1}{2}} \quad (10)$$

In the logistic model, d denotes the distance between the errors, which also represents the distance between the observations from the estimated regression line.

Pergibon (1981) presented a robust method for estimating regression parameters, which can be calculated by minimizing them. In the problem of profiles, the proposed method of calculating the function is expressed as follows:

$$L(\beta) = log\, \rho\left(d^2\left(\mu_{y_i}, y_i\right)\right) \quad (11)$$

where ρ an ascending function with a slope lower than the identity function. Crooks and Hisbrook (2003) showed a function that minimizes the above equation ($\varphi = \rho'$)

$$\varphi_0(u) = exp\left\{-max\sqrt{(u, c)}\right\} \quad (12)$$

Deriving $c > 0$ from Equation results in the following formula:

$$\sum_{i=1}^{n} \varphi(d_i^2(\beta))\left(y_i - \mu_{y_i}\right) \quad (13)$$

Where $d_i(\beta) = d\left(\mu_{y_i}, y_i\right)$. the estimates are derived by zeroing Equation 13.

$$\sum_{i=1}^{n} \varphi(d_i^2(\beta))\left(y_i - \mu_{y_i}\right) = 0 \quad (14)$$

As such, the β that zeros Equation 14 is the desired solution for parameter estimation.

3. Weighted Ridge M-estimator method

This method, which is a combination of the previous two methods, was presented by Crooks and Hisbrook (2003) to improve the efficiency of the ridge M-estimator method. From Equation 11:

$$L(\beta) = \sum_{i=1}^{n} w_i\left(\rho\left(d^2\left(\mu_{y_i}, y_i\right)\right)\right)$$

$$\rho(t) = \begin{cases} t.exp\left(-\sqrt{c}\right), if: t \le c \\ -2\, exp\left(-\sqrt{t}\right).\left(1 + \sqrt{t}\right) + exp\left(-\sqrt{c}\right).(2(1 + \end{cases} \quad (15)$$

where the value of w_i is obtained from the Eq. 7. Now Eq. 15 should be minimized using the same methods as Eq. 1 to calculate β values.

The case selected for this research is the case formulated in Yeh et al (2009), where X (vector of independent variables) has 9 levels:

$$X = \begin{bmatrix} log(0.1), log(0.2), log(0.3), log(0.4), log(0.5) \\ log(0.6), log(0.7), log(0.8), log(0.9) \end{bmatrix} \quad (16)$$

here, m and k are assumed equal to 30. That is, a response variable with a Bernoulli coefficient of k = 30 is used, and given that a binomial distribution is used, m = 30 profiles are generated in each stage of the simulation. The lower logit interface function is used to generate data:

$$\pi_i = \frac{e^{3+2x}}{1 + e^{3+2x}} \quad (17)$$

The authors of the current study have sought to propose novel methods for estimating the parameters of the logistic-regression profile, for the monitoring of which the T^2 method is employed. 5 methods presented by Yeh et al. (2009) were reviewed and only the most effective method, T_I^2 , was represented here. The proposed methods for expanding simple linear profiles are presented and the estimation methods are examined for these profiles.

Findings

1. Shift in several levels of profiles

In this scenario, $\beta = (\beta_0, \beta_1)$, $\beta_0 = 3$ and $\beta_1 = 2$. First, the effect of the incremental shift of parameters using 4 methods of maximum likelihood estimation (MLE), weighted maximum likelihood estimation (WMLE), ridge m-estimation (RM), and weighted ridge estimation (WRM). The shifts are effectuated in the 5th, 10th, 15th, 20th, 25th, and 30th profiles. This has been done for 10,000 simulations.

Tables 1 and 2 respectively present the mean and standard deviation for the estimation of the parameter β_0 with a shift $\beta_1 + \delta$ for different δ values using the estimation methods. Also respectively present the mean and standard deviation for the estimation of the parameter β_1 with a shift $\beta_1 + \lambda$ for different λ values using the estimation methods mentioned.

Table 1: Mean values for the estimation of parameter β_0&β_1

δ	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**
MLE	3.0020	3.0659	3.4557	3.8452	4.2314	4.1312
WMLE	3.0154	3.0220	3.0765	3.1634	3.4621	3.9545
RM	2.9870	3.0056	3.0124	3.0865	3.1478	3.3502
WRM	3.0103	3.0051	3.0102	3.0666	3.1014	3.2886
λ (β_1)	0	0.3	0.6	0.9	1.2	**1.5**
MLE	2.0018	2.0244	2.1516	2.4573	2.8455	**3.0142**
WMLE	2.0174	2.0088	2.0277	2.1014	2.3321	**2.8851**
RM	2.0287	2.0231	2.1131	2.1595	2.2704	**2.3025**
WRM	2.0289	2.0326	2.0226	2.0481	2.0611	**2.1873**

Table 2: Standard deviation values for the estimation of parameter β_0&β_1

δ	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**
MLE	0.0016	0.2425	0.3038	0.6801	0.7704	0.7885
WMLE	0.1044	0.0233	0.0336	0.1020	0.1248	0.3015
RM	0.1186	0.0173	0.0194	0.0313	0.0842	0.1796
WRM	0.1200	0.0113	0.0147	0.0224	0.0678	0.0911
λ **(β_1)**	0	3	6	9	1.2	1.5
MLE	0.1045	0.1158	0.3278	0.3758	0.5735	0.5985
WMLE	0.2341	0.1516	0.0457	0.1621	0.1245	0.1125
RM	0.2578	0.1135	0.0385	0.0311	0.0955	0.1138
WRM	0.2475	0.1013	0.0521	0.0479	0.0714	0.0983

Findings from the Tables indicated that the MLE method is strongly affected while shifting the parameters. The shifted parameters only partially affect the robust-based methods, and the WMLE method outperforms its non-weighted counterpart, i.e., MLE. Among the three proposed robust methods, the WMR method produces the highest efficiency and exhibits less susceptibility.

Next, type I and type II errors from step changes in the model parameter are evaluated. The upper control limit (UCL) T_I^2 is equal to 15.0786. Tables 3 presents show the values of type I error and type II error (II), respectively for the shift in parameter β_0 and β_1

Table 3: Type 1 error and test power (both) for the shift in parameter β_0&β_1

δ	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**
MLE	0.0501	0.0560	0.0665	0.0731	0.1891	0.1015
WMLE	0.0496	0.0588	0.0755	0.0911	0.1088	0.1375
RM	0.0604	0.0790	0.0754	0.0824	0.1117	0.2289
WRM	0.0501	0.0587	0.1798	0.0885	0.0169	0.1237
λ **(β_1)**	0	0.3	0.6	0.9	1.2	1.5
MLE	0.0501	0.0568	0.0655	0.0756	0.0765	0.0857
WMLE	0.0504	0.0590	0.0769	0.0882	0.0996	0.1097
RM	0.0503	0.0598	0.0770	0.0887	0.1030	0.1146
WRM	0.0513	0.0606	0.0808	0.0946	0.1079	0.1200

2. Step change

Here, the data is generated in a controlled manner, and the parameters suddenly change after a particular point. That is, the 1st to (k-1) th profile are generated with $\pi_i = \frac{e^{\beta_{0,in}+\beta_{1,in}}}{1+e^{\beta_{0,in}+\beta_{1,in}}}$ and profiles k to m are generated with $\pi_i = \frac{e^{\beta_{0,in}+\beta_{1,in}}}{1+e^{\beta_{0,in}+\beta_{1,in}}}$. $\beta_{0,in}$ and $\beta_{1,in}$ are the controlled values for the parameters of the binary equation, which are assumed to be equal to 3 and 2, respectively. Furthermore, $\beta_{0,out}$ and $\beta_{0,out}$ are out-of-control values for independent and dependent parameters of the logistic profile, which are calculated as follows for a given step change of for step shift $\delta + \lambda$:

$$\beta_{0,out} = \beta_{0.in} + \delta \tag{18}$$

$$\beta_{1,out} = \beta_{1,in} + \lambda \tag{19}$$

Here, by giving an example with a different value of k, we show the test power of the proposed methods. We set the number of profiles equal to 30 and select the shift to apply to the parameter. When k takes the two values of fifteen and twenty, table (4) shows the power of the test of different methods.

Table 4: Test power values for step changes in parameter β_0

	δ	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1.1**
K = 15	MLE	0.0530	0.1509	0.3410	0.6645	0.8146	**0.9**
	WMLE	0.0536	0.1688	0.3691	0.6858	0.8627	**0.8239**
	RM	0.0441	0.1636	0.3863	0.7142	0.9013	**0.9885**
	WRM	0.0646	0.1850	0.4100	0.7136	0.9348	**1.0001**
K = 20	MLE	0.0543	0.1726	0.3928	0.6805	0.9357	**0.9550**
	WMLE	0.0542	0.1779	0.4024	0.7111	0.7997	**0.9974**
	RM	0.0548	0.1851	0.4317	0.7214	0.8116	**1.0001**
	WRM	0.0543	0.1878	0.6354	0.7405	0.9825	**1.0250**

The following results are obtained for β_1.

Table 5: Test power values for step changes in parameter β_1

	λ	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
K = 15	MLE	0.0535	0.2624	0.3555	0.6575	0.8258	**0.9125**
	WMLE	0.0559	0.0778	0.3875	0.6996	0.8769	**0.9264**
	RM	0.0541	0.2819	0.3999	0.7049	0.9112	**0.9980**
	WRM	0.0539	0.0889	0.4112	0.7156	0.9358	**1.0003**
K = 20	MLE	0.0557	0.0821	0.4001	0.6811	0.85111	**0.9572**
	WMLE	0.0565	0.0873	0.4116	0.7030	0.8873	**0.9951**
	RM	0.0563	0.2902	0.4145	0.8129	0.9331	**1.0070**
	WRM	0.0571	0.2994	0.4335	0.8226	0.9818	**1.0003**

Findings from Tables 4& 5 indicate that for step changes, the common MLE method strongly reacts to half of the profiles and increases their parameters. The results further indicate that changing the β_0 results in a type II error hitting 1, which reveals the extreme weakness of this method.

It is clear that robust methods have more acceptable efficiency compared to the MLE method, that among the three mentioned methods, WRM shows the most acceptable efficiency in such a way that if 10 profiles are shifted and their β-1 parameter is from 2 to 6. 2 to increase, the second type error is about 0.3, which is a huge difference compared to the common method, which is slightly more than 0.9.

The result of comparing the methods is shown below:

$$MLE < WMLE < RM < WRM$$

3. Drift

In drifts, the data increases gradually and with a constant trend after profiling. That is,

$$\begin{cases} \beta_{0,t} = \beta_0 + r.(t-k) \\ \beta_{1,t} = \beta_1 + r(t-k) \end{cases} t = k+1, k+2, \dots, m \tag{20}$$

where r is a constant, and t is the profile after which β values have shifted or changed. For our case study, t has been assumed to be 10 and 15 and r is assumed to be 0.05. In other words, this shift starts from the 11th profile and continues until the 30th profile. m is the number of profiles and is equal to 30.

Table 6 presents the effect of this shift with t=10 on the test power by changing the parameters in the 4 proposed methods.

Table 6: Test power for drift of (t=10)

Estimation method	**Change in β_0**	**Change in β_1**

MLE	0.9424	**0.9864**
WMLE	0.9556	**0.9867**
RM	0.9814	**0.9985**
WRM	0.9902	**1.000**

Figure 1 plots the effect of drift with $t = 15$ on the test power with a change in parameters β_0 and β_1. Each estimation method is specified by a parameter in the figure.

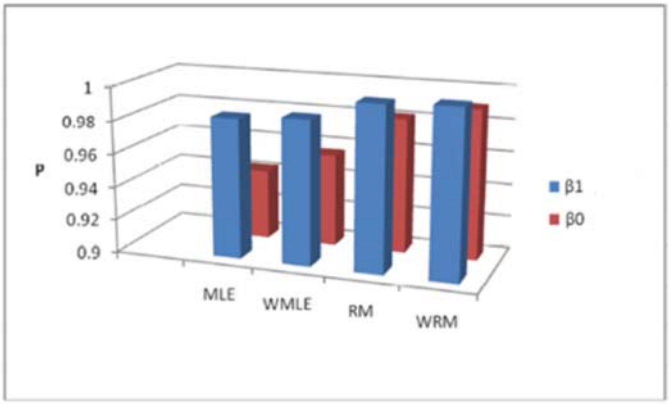

Figure 1: Test power plot for drift in t = 15 s

the MLE method is the most manipulable method, which is susceptible to the greatest share of errors. Moreover, the robust methods exhibit far greater stability against drifts of the parameters and a lower error rate compared to the classical method. Ultimately, the results show that regardless of the change in the parameter level and the number of profiles, the WRM method produces far better performance than other methods.

Conclusion

In this research, the numerical examples used by Yeh et al. (2009) were used to evaluate the effect of autocorrelation on the monitoring of logistic profiles. The results indicate that in the presence of outlier data, the type I error rate is highly inflated. Furthermore, the proposed calculation methods and their results were tested on step-changes and drift. Similarly, a test power diagram was plotted for each of the proposed methods. The findings indicated that the robust-based methods significantly outperformed the classical methods in terms of step-change and drift. Among the proposed robust methods, the weighted ridge M-estimate combined method exhibited the best performance, while the MLE method performed the worst among the four proposed methods. The findings further revealed that the MLE method is the most manipulable method, which is susceptible to the greatest share of errors. Moreover, the robust methods exhibit far greater stability against drifts of the parameters and a lower error rate compared to the classical method. Ultimately, the results show that regardless of the change in the parameter level and the number of profiles, the WRM method produces far better performance than other methods.

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Amiri, A., Ayoubi, M., Haji Seyed Razi, S. A. (2011). Improving statistical process monitoring with qualitative characteristics in the form of polynomial profiles in phase II. Production and Operations Management 2(2): 73-83.

Taghipour, M., Fathi, K., Nikoyifar, A., Falahdoost. F. (2015). Monitoring of multivariate linear profiles in phase 1. The first international conference on industrial engineering, management, and accounting.

Hakimi, A., Amiri, A., Kamran-Rad, R., (2018). A robust method for monitoring logistics profiles in Phase I. Production and Operations Management 9(1): 107-118.

Soleimanian, M. E. (2011). Development of monitoring methods of profiles with the binary response variable. Master thesis, the Sharif University of Technology, Faculty of Industrial Engineering.

Noorosana, R., Amiri., A. (2007). Improving the monitoring of linear profiles in phase II. Amirkabir 18 (66): 19-28.

Pignatiello, J.J, and T.R Samuel. 1998. "Identifying the time of a step change with X control charts." Journal of Quality Engineering 10 (3): 521-527.

Shadman, A, C Zou, H Mahlooji, and AB Yeh. 2017. "A change point method for Phase II monitoring of generalized linear profiles." Communications in Statistics-Simulation and Computation 46 (1): 559-578.

Shadman, A, H Mahlooji, A Yeh, and C Zou. 2015. "A change point method for monitoring generalized liner profiles in Phase I." Quality and Reliability Engineering International 31 (8): 1367-1381.

Sharafi, A, M Aminnayeri, and A Amiri. 2012. "Identifying the time of step change in binary profiles." The Int. J. Advanced Manufacturing Technology 63 (1-4): 209-214.