

## Deterministic methods for solving the N\_Queens problem, a strategy for proving $p=np$

### Abstract

Computational Complexity theory (NP) is a branch of the Theory of Computation, mathematics, and computer science that deals with the difficulty of solving problems algorithmically. This research is trying to focus on one of the representative problems of artificial intelligence called N\_Queens by presenting some suggested ideas to help solve the  $np\_complete$  problems to find the above algorithmic structure with high speed by providing suitable algorithms to solve the N-Queens problem. In the N-Queens problem, the goal is to place chess queens on an  $n \times n$  chessboard so that no two queens are on the same row, column, or diagonal. The N-Queens Completion problem is a variant, dating to 1850. This technique can most likely be used to solve the intractable problems of artificial intelligence. The n-Queens puzzle is similar to another problem called the P versus NP problem in computer science. All well-known intractable problems that are very practical and important can be solved by solving  $P=NP$ .

**Keywords:** *Reduction,  $P=NP$ , N-Queens problem,  $np\_complete$*

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### Introduction

The complexity class P is one of the members of NP, but NP includes other important classes [1, 2], the most complex of which is NP-Completion, such that no known algorithm for them can be executed in polynomial time. In the chess game, the chess queen can move horizontally, vertically, and diagonally. The N-Queen's problem is to place n chess queens on an  $n \times n$  chess board so that no two queens threaten each other. The first n-queen problem was presented in 1848 called the Eight Queens puzzle. Franz Nauck published the first solutions for Eight Queens in 1850. He generalized the problem of the Eight queens puzzle to the problem of n queens, which is the arrangement of n queens on an  $n \times n$  chess board so that they do not threaten each other. Many mathematicians worked on the problem of eight queens and its generalized state or the same n-queens and tried to solve it (such as Karl Friedrich Gauss). Gunther (1874) presented a definitive method to solve this problem, and Glaisher improved this procedure. Finally, Dijkstra (1972) presented the Depth-First Backtracking algorithm to solve this problem. Solving the Eight Queens puzzle is mathematically very expensive because, in only one  $8 \times 8$  page for eight queens, there are 368,165,426,4 possible positions on the chessboard, of which only 92 positions are possible to solve the puzzle. The 12 solutions are unique. It means the rest of the answers are obtained from the symmetry of the original answers. Let's that the computational complexity increases with the n value increase. In this regard, methods have been presented to reduce the computational complexity to solve this problem. Finally, we will get the  $n!$  the possible situation that it is an exponential problem. In general, the n-Queens problem is considered one of the  $np$  problems in artificial intelligence. Cook's theorem (1971) proved that  $P = NP$  if it is verified that the satisfiability

belongs to the P class, and vice versa. Acknowledging this, the satisfiability problem is of great importance. The Monte Carlo algorithm is used to estimate the efficiency of a backtracking algorithm. Monte Carlo algorithms are probabilistic, that is, the next executive order is sometimes randomly determined. This does not true for the deterministic algorithm. The Monte Carlo algorithm estimates the expected value of a random variable defined on a simple space using its mean on a random sample from the simple space [4]. Backtracking [5] is a technique in which all possible situations are developed, and every developed situation is checked to see if it is right or wrong, and if it is wrong, it develops another situation and the same process continues. In the research literature [6, 7], we witness the repetition of specific patterns of chess pieces in the queen problem, which are limited and selective and do not include all dimensions of the problem. Therefore, the discussion of reduction [8], is based on a logical relationship based on whether it is true or false, regardless of calculations and mathematical proof and theorems raised in this matter. It is a solution through which other  $np\_complete$  problems such as 3sat and k\_clique can be solved by reducing  $np\_complete$  problems to each other.

The n-Queens problem is not a scientific puzzle, but a scientific problem and  $np\_complete$  in mathematics and computer science and as a criterion in artificial intelligence. Reducing and solving this problem can be the solution to thousands of other unsolved scientific problems, through which we can hope for the realization of the  $p=np$  problem equation.

The titles raised in this research refer to the process of designing and solving the N-Queens problem with creative methods. A logical result can be obtained by solving the proposed techniques and finding a definitive answer sequentially or a combination of two random and sequential

techniques. Therefore, a forecasting method was used to predict the possible answers or to identify Hopeful homes and critical points. In the rotational rule and the fractal pattern, it is possible to examine a suitable pattern with the reverse approach, rotation, repetition, and the development of different situations of the problem. The reflexive rule refers to the diagonal reverse pattern and in the quadratic progression rule, all the answers follow a pattern or square of  $n^2$  numbers. But in the quarter rule, which is a quarter of the problem, we can identify all the answers in one problem quarter.

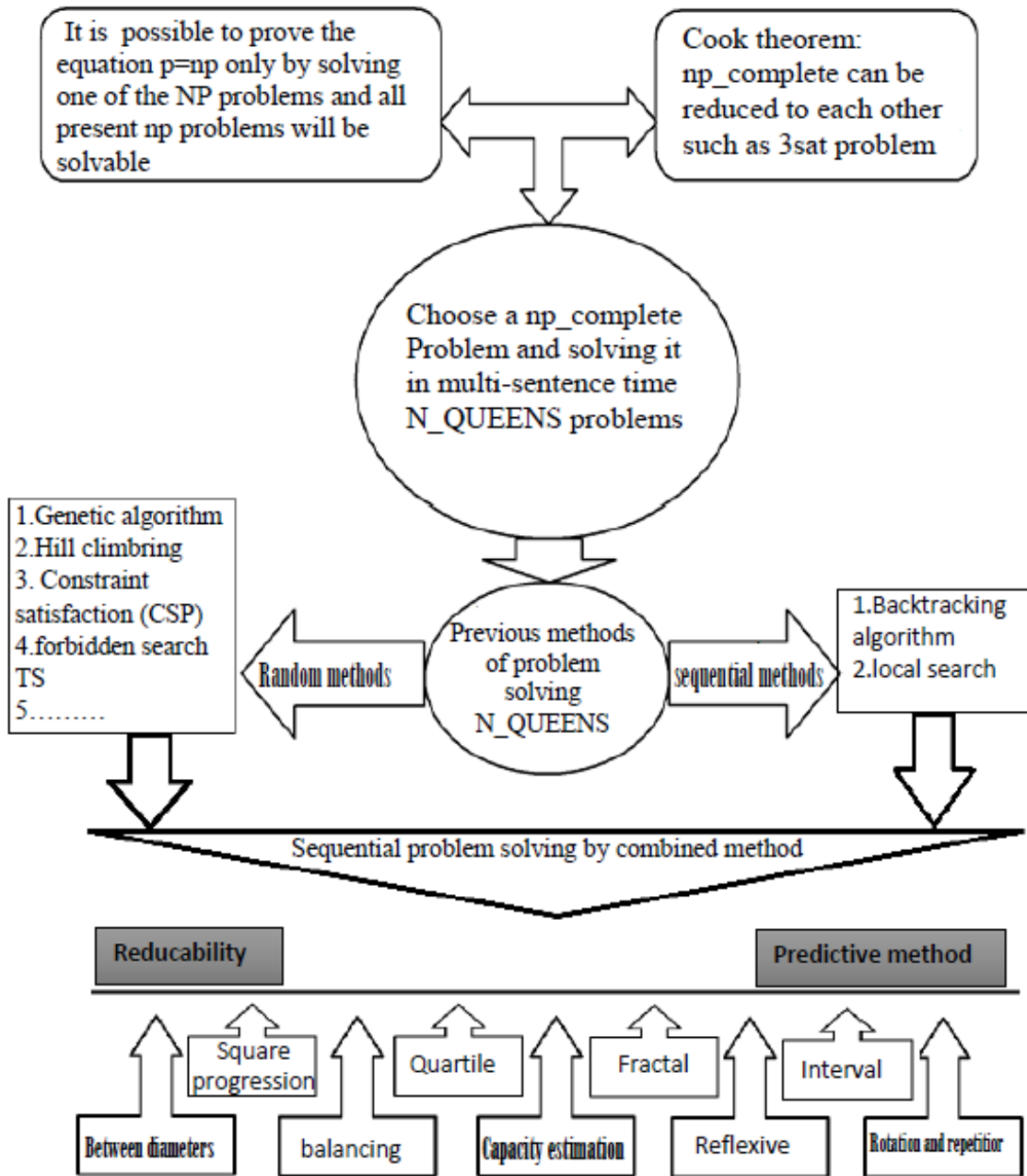
The current solutions move randomly and non-deterministically in the path of solving problems. The solution to all the problems in this field should be provided not only by the random method but by the deterministic (sequential) solution in simple language of the N\_Queens problem and reducing it to other intractable and sat3 problems and limiting the computational space, focusing on the sequential and non-random problem-solving. In this research, various innovative techniques were used, including the reflection method, fractal method, Interval arithmetic, limit method, rotational method, balance ability, quadratic progression, quadrant, capacity estimation, reducibility, interdiagonal, and predictor, which are

presented in the statistical and algebraic form of numerical and probabilistic progression.

The goal of solving the n-Queens of chess problem is to arrange n chess queens on an (n×n) chessboard so that no two chess queens are in the same row, column, or diagonal. The chess queen can move horizontally, vertically, and diagonally. The n- Queens puzzle is one of the NP problems in artificial intelligence that conventional search methods will not be able to solve.

### **Methodology**

A specific framework for the proposed method is presented to improve the performance of the considered algorithm.



**Figure 1. Outline of the conceptual model of the proposed design**

In the present research, several innovative methods of solving the n-Queens problem (to reduce the complexity of the n-Queens problem from the  $N!$  situation) were used. As a result of solving this problem, we can provide several suitable solution methods to achieve the goals of this scientific research, i.e. solving and proving NP-complete problems and finally solving the  $P=NP$  problem.

In innovative methods, it was tried to discover a movement pattern between the n-Queens.

In the conducted surveys, we can point out several points that are of particular importance:

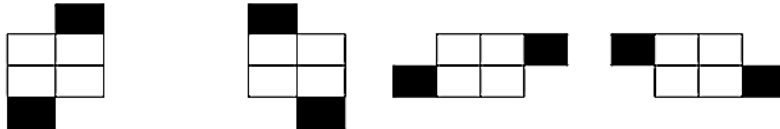
1. Solving the diagram problem according to the movement method of the chess pieces and discovering and predicting and placing them correctly.
2. All the chess pieces follow an appropriate movement pattern that is repeated regularly in the process of discovering the next chess pieces.
3. All the chess pieces should be placed with the least error and the highest predicted probability according to numerical calculations.
4. The ability to implement diverse and creative methods of solving problems by combining sequential and random methods.

- 5. The possibility to compare and algebraic calculation of methods  $a$ ,  $\bar{a}$ .
- 6. The suggested techniques can always have correct relationships for correct answers, but not always for wrong answers.

**Rules of problem-solving**

**1. Reflexive rule**

All the chess queens on the chess board have a reflective relationship with each other forever, which behaves

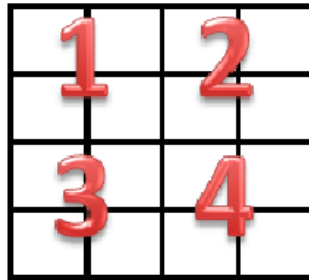


**Figure 2: The rule of square exponential growth with different n values**

According to this principle, it can be said that the process of growth and movement of chess pieces between rows and columns will be exponential for more n as (Figure 3) which can be seen in the four dimensions (Quarter 1-Quarter 2-Quarter 3-

exponentially and as a square relationship. This rule can be valid from 1 to n number of chess pieces. The square progression rule, which can be shown as  $n^2$  has this relationship with each other collectively. This rule applies to the first and second half of the table and all the first, second, third, and fourth quadrants. The chess pieces have a reflective relationship with each other and can change and rotate at different angles (90, 180, 270, and 360) (Figure 2).

Quarter 4) similarity and a reflective relationship. Each quadrant will have a reflective relationship with other quadrants in the row (quadrant 2) and column (quadrant 4) and in the diameter (quadrant 3), while  $\frac{1}{4}$  table forms a quadrant.



**Figure 3: Showing the relationship of the quadrants of a table**

According to the reflexive rule of chess pieces, it can see the relationship between the coaching relationship between the first and third quadrants as well as the second and third quadrants. This relationship will help in designing a suitable and efficient algorithm.

One can witness a beautiful discipline throughout the issue of that n-queens, considering the order governing the tables (Figures 5 and 4) in such a way that these relationships are repeated many times between all the chess pieces. It can observe the squareness, reflection, and symmetry in relation to each other. There is a relationship between square progression and reflection and symmetry. As you can see, the chess piece

can be seen in the first circle with the color (pink), if it is in the (1.1) square (Figure 4). The proportional trend of the movement and growth of the 8 path (blue color) was observed in the first row of the table, which was classified into two parts, the upper half and the lower half (top of Figure 4) symmetrically in (bottom of Figure 4). This situation is repeated in the next rings, i.e. the second ring (green), the third ring (orange), and the fourth ring (red). Interestingly, all rings will keep their position relative to the assigned square if another square is selected instead of the (1.1) square and they will adapt to the selected square. Accordingly, each row will have a maximum of four selection modes relative to the specified square.

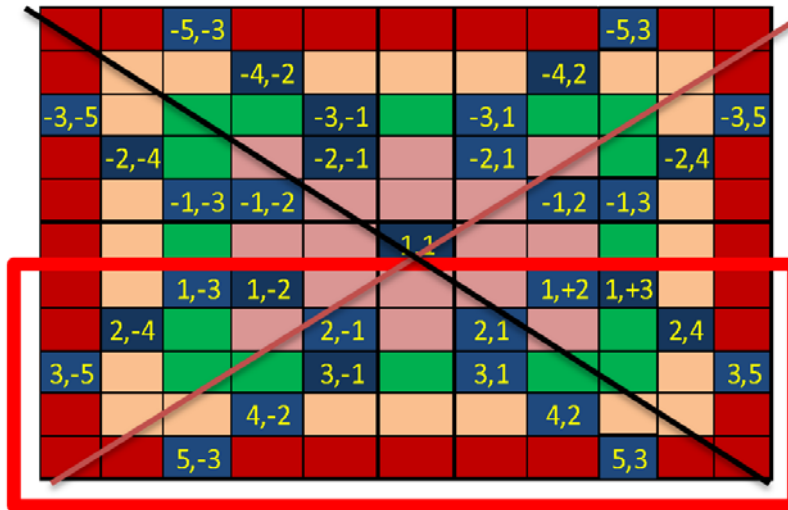


Figure 4: schematic of the exponential growth of squares and surrounding rings around the axis of the square (1×1) which is created in eight directions and four quadrants.

	s1	s2	s3	s4	s5	s6	s7	s8
1	-1,-2	-2,-1	-2,+1	-1,+2	+1,+2	+2,+1	+2,-1	+1,-2
2	-1,-3	-3,-1	-3,+1	-1,+3	+1,+3	+3,+1	+3,-1	+1,-3
3	-2,-4	-4,-2	-4,+2	-2,+4	+2,+4	+4,+2	+4,-2	+2,-4
4	-3,-5	-5,-3	-5,+3	-3,+5	+3,+5	+5,+3	+5,-3	+3,-5
5	-4,-6	-6,-4	-6,+4	-4,+6	+4,+6	+6,+4	+6,-4	+4,-6

Figure 5: Schematic of the relationship of 5 rings from a table which can be seen numerically, symmetrically, and reflectively for the square (1×1)

## 2. The rule between diagonals

In the following, we will discuss how to place the diagonals in the table with  $n = 6$ , so that the main diagonal (right side) is marked with zero value and the secondary diagonal (left side) is marked with seven of 7. In the right half of the table, we can see the result of subtracting the selected values (marked at the top of the table from 1 to 6), which are normally placed in the form of consecutive zeros along the right diagonal and the sum of the selected values as specific values of  $n$  is on the left side of the table, which is 7. The values in rows (1 to 6) of the table are a regular representation of the default sketch of the main (right) and accessory (left) diagonals of the 6x6 table, and the diagonals are seen repeatedly and diagonally in the table. As we mentioned, the following code snippet refers to the

relationship between the diameters, it refers to each other when the chess pieces collide with each other. Lets  $|col(i)-col(k)| = |i-k|$  If two queens are in one column then  $col(i) = col(k)$  and if two queens are in one row.

If  $((Col(i) - col(k) == i - k \text{ OR } if (Col(i) - col(k) == k - i))$

It should be noted that the condition of not colliding the chess pieces will be the following three clauses:

if two ministers are in the same column

i)  $j=l$

Checking the main diagonal (right side)

ii)  $i - j = k - l$

Checking the accessory diagonal (left side)

iii)  $i + j = k + l$

	1	2	3	4	5	6		1	2	3	4	5	6	
27	2	3	4	5	6	7	1	0	-1	-2	-3	-4	-5	-15
33	3	4	5	6	7	8	2	1	0	-1	-2	-3	-4	-9
39	4	5	6	7	8	9	3	2	1	0	-1	-2	-3	-3
45	5	6	7	8	9	10	4	3	2	1	0	-1	-2	3
51	6	7	8	9	10	11	5	4	3	2	1	0	-1	9
57	7	8	9	10	11	12	6	5	4	3	2	1	0	15
252	27	33	39	45	51	57	21	15	9	3	-3	-9	-15	0

Figure 6: A different schematic from the 6x6 table with the main and accessory diagonals with the values of the subtractions on the right side and the sums on the left side. These values of the problem of verification of solutions will be achieved by reflection and non-repetition.

the lower half of the table with the values 1, 3, and 5 with the same situations of at least one distance apart (right side of Figure 7). But it can be seen in the rows differently that by rotating the table by 90 degrees, it is considered just like the columns because the order of operations is not based on the table index. But in the following (right side of Figure 7), the distance between the chess pieces compared to the main and accessory diagonals is presented according to the quadrants of the problem-solving table, repeating the symmetry and reflecting a specific ratio between them, which can be calculated.

### 3. Interval rule

In the Interval rule, the problem states the minimum distance between the answers in the rows and columns and the diagonals of the problem solution, so there must always be one or more distances between the correct and minimum answers. The 50% of the answers should always be in the upper half and the other 50% should be in the lower half of the table. For example, the columns in the upper half of the table such as the values 2, 4, and 6 with at least one space can be a suitable answer, and in the same way, they are placed in the columns in

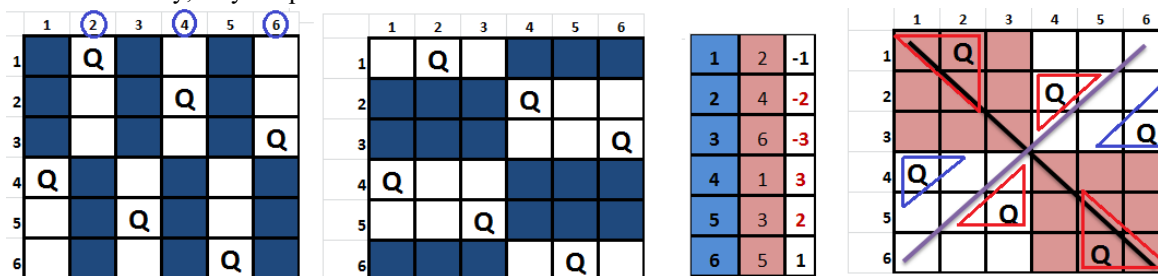
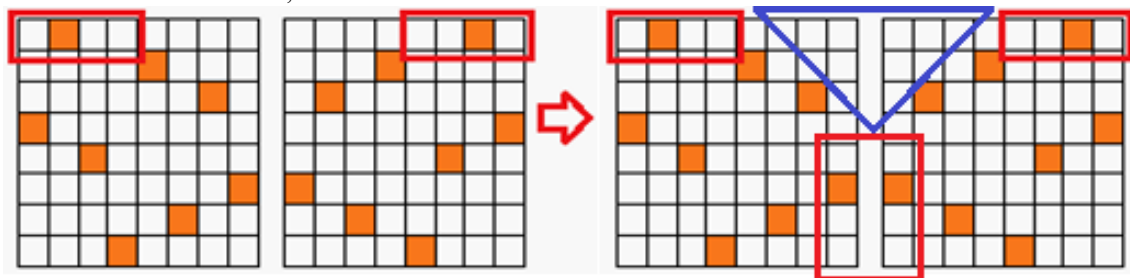


Figure 7: Placement modes of stamps by observing the distance and placement positions of the chess pieces in the columns and rows and diagonals of the table

repeatability of the problem symmetrically (folding the table), it is enough to solve the  $n/2$  steps of the problem. This means navigation (Figure 8).

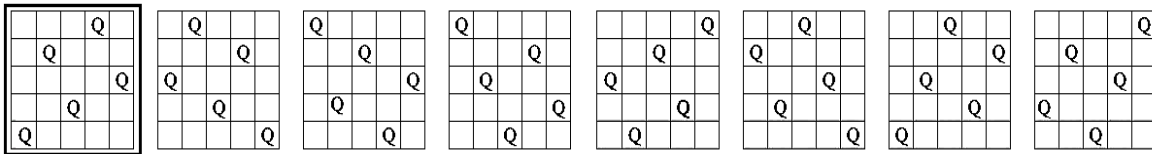
### 4. The repetition and rotation rule

In the first row of the problem-solving table, there will be no need to solve the whole 1 to n value, because due to the



**Figure 8**

Up to 50% of the problem-solving. In general, the focus should be on single and main solving problems to reduce the complexity and avoid similar repetitions (in the 8×8 problem, we will have 12 main unit answers and 92 repeatable and rotatable non-unit answers). For example, it is enough to survey the problem in the first row to column 4 if n=8 and it is a repeated reflex: The left side of the figure refers to the first row of the table, that is, solving problems of the first half of the

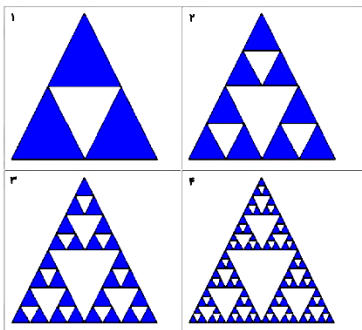


**Figure 9: Schematic of a 5×5 table with a unique mode and 7 repeatable rotation modes**

**5. The rule of fractal patterns**

Fractal patterns derived from nature is a geometric structure that repeats the same initial structure by expanding each part of this structure to a certain ratio. Modern network systems (categories of drones, social networks, network production chains, transportation and logistics networks, communication networks, and cryptocurrency networks) are multi-elemental, all placed in fractal patterns. This means that it is possible to reach a suitable rule with a little rotation and create symmetry or use the solution of previous problems and similar aspects. For example, for the Eight Queens puzzle, the mentioned items are significant:

1. There are several possible states for the position of 1 chess queen, 2 chess queens, and 3 chess queens in a quadrant of the 4×4 table for n=8 to comply with the  $n^2$  principle of square progression.
2. How many times and with what states are the positions created and repeated in different quadrants of the table?
3. Possible positions for different values for existing balances can be created and stored to be repeated and used in larger dimensions.

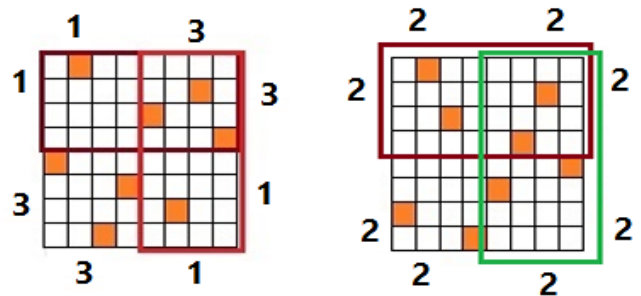


**figure 10: An sample of fractal geometric representation based on a repeating triangle pattern**

table are repeated symmetrically. On the right side of the figure, with the pattern of the square progression rule, there is no need to calculate the triangular range from columns 5 to 8 of the problem. The rotations are 90, 180, 270, 360, folding, reflective, and diagonal, so in Figure 9, we can see seven different states of a different correct answer from different angles.

**6. Balancing rule**

We have an interesting relationship in the analysis of different modes of problem-solving that to find a suitable answer in problem-solving, a suitable balance must be reached between the four quadrants of the problem-solving table, which is used as a generic rule. It is interesting here that the rule can be used for all dimensions of problem-solving so that this method can be used more in subsequent analyzes (such as the quartile technic).



**Figure 11: Schematic of balancing in a table with two states of equal values and difference with 2 value**

Here, the relationships between the four quadrants of the table (Figure 11) were used in solving the problem, and a balance was achieved so that a rule could be realized through it. This means that there are two types of relationships between each quadrant of the table and the other quadrants of the problem. We find an equivalence relation or an equal position with zero difference among the correct answers to solving the problem (right side of Figure 11). In solving the problem, we will get an unequal relationship with a difference of two orders, that is, a difference of two in some correct answers, according to the equations  $2-2=0$  and  $1-3=2$  (Figure 11, left).

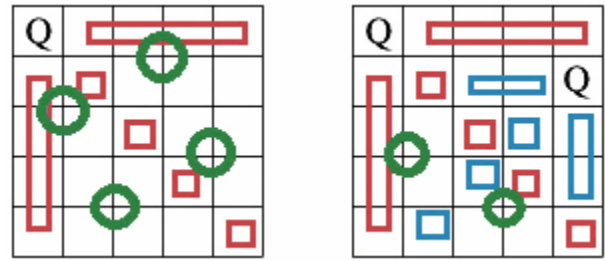
**7. Capacity estimation rule**

The purpose of estimating the detection capacity is to continue solving the problem and get the correct answer with an exact estimation percentage (Figure 12) so that the minister must have the necessary and sufficient capacity to solve the problem for squares of the chess queens. It means that it is possible to continue solving the problem according to the problem-solving rule. Squares are removed from the selection list when the chess queen is selected, and based on the existing relationships among the squares, it can be estimated whether there is the possibility of the necessary capacity to continue the problem or not.

Each chess queen has a limit in the surrounding rings, which from the first ring is limited to eight neighboring squares, but each chess queen has its minimum limit of  $n=2$ , i.e.  $(2 \times 2)$ .

In both rows, we have 4 squares in common, and it will be considered in the estimation list if one of them can be a candidate. The estimation of the squares starts with the selection of the first chess queen and the necessary capacity to continue the path of the second chess queen candidate, and the second estimate is done twice for the second chess queen candidate. It should be noted that each circle or ring can cover up to 4 squares and it can be our next choice if one of the 4 squares can be used. It is better not to place the estimation rings in the same row, column, and diameter for a better result. In this way, we exclude the range of each chess queen with at least one distance around it, which is considered a critical point in any position, from the calculations circle. We will check the solution to the problem with a survey.

This problem can continue without coming back and by going through the path once, that is, it completes the solution of the problem if the estimation fails. It means that a mathematical relationship established in the rule of problem-solving can be used appropriately to make a more accurate estimate.



**Figure 12:** The estimation scheme of a 5x5 table for capacity measurement to continue solving the problem, which is marked with green circles. Each green circle is characteristic of choosing a square with a critical probability of 1 to 4 squares.

For example, the 5x5 table in the estimation of the first choice in the first row makes it possible to choose the next four squares, but in the second choice in the second row, which is square 5, it cannot be a suitable estimation, because the squares selected by the circles are only two of the squares that are paired together. As a result, in the example on the right, there is no capacity to select 3 chess queens in the selected range. Therefore, continuing to solve the problem by estimating the selection of two ministers will not be possible and will fail.

**8. Quadrilateral rule**

In the quadratic rule, we convert the queens' puzzle, which is divided into a table with four equal parts of quadrants 1 to 4, into a smaller table equal to a quarter of the original table (Figure 13). The purpose of this operation is to discover new relationships between chess pieces, which has a significant impact on better solving the problem. We will see interesting relationships between chess pieces if we pay attention to the process of changing or compressing the problem to a quarter. In the columns of the quartile table, there are always constant values, the sum of which has a difference of only 2 values, and this difference, which is indicated by the 2 value, is true in all aspects of the Queen Problem.

								5	6	7	8									5	6	7	8																	
								1	2	3	4									1	2	3	4																	
1	1													0	1	1														5	1	1			3					
2					5									-3	2	5														6	2	5			7					
3													8	-5	3							8								7	3				2				8	
4						6							-2	4		6														8	4			6				4		
5			3									2	5			3														6			8	10	12					
6								7				-1	6			7																								
7		2										5	7		2																									
8				4								4	8				4																							
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**Figure 13: Showing how to create a quadrant table using the four quadrants of the table**

**9. square progression rule**

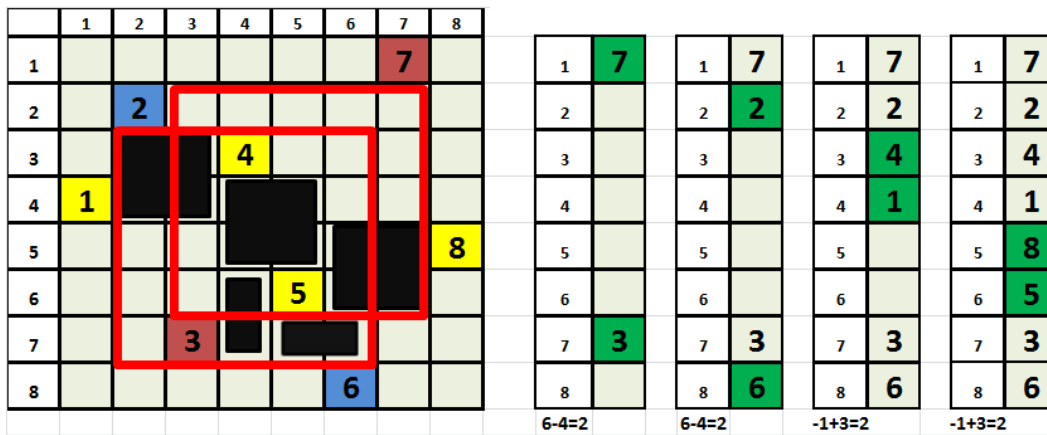
According to the reflective rule (Figure 2), each chess piece must have at least a square relationship with other selected chess pieces that can grow exponentially using the  $n^2$  principle.

In this technique, it is possible to identify the selectable values with the least number of occurrences through each selection that is made, that is, each selection number has a square relationship  $n^2$  with the candidate's value.

1,7 7,3 6,4 6-4=2	1,7     1,7 2,5     3,8 1,2     2,1 2-(1-1)=2	1,7     1,7 4,6     2,4 3,1     1,3 3-1=2	1,7 4,2 3,5 5-3=2	1,7     1,7 3,3     7,3 2,4 4-2=2	1,7 7,3 6,4 6-4=2
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**Figure 14: How to calculate methods of square progression and transformation and prove problem correctness**

The next selection of details (rows and columns) ends with the number two. For more details, see the figure below.



**Figure 15: How to arrange selected chess pieces and establish  $n^2$  relationship between them**

According to the example, if our first choice is number seven, we can offer specific answers with appropriate coefficients (Figure 15). For example, in the second row (2.5) and (2.4) and the third row (3.8), (3.6), and (3.3), the chess queen can be a candidate for selection, but we assume that these values were selected in the previous stages and to avoid repetition of values, we search for the newer choices. As it has been said, the value

of the square of the candidate selected by the square (1 and 7) is the square (3 and 7). It means choosing the first candidate square that uses the  $n^2$  principle, where  $n=5$  and uses a  $5 \times 5$  square. Each  $n^2$  can be effective in square progression (table 1). As you can see in the example, there is only one exception, and that is when the value of  $n=1$ , appears to be a rectangle, not a square, and this will not violate the  $n^2$  principle.

**Table 1: How to calculate the quadratic progression method and the growth trend and the general rule between them.**

n	1	2	3	4	5	...n
$n^2$	1	4	9	16	25	.....

**10. Reducibility rule**

Many papers have been presented on the reducibility of the  $n$ \_queens problem to other  $np$ -complete problems and the proof of their  $np$ \_completeness. Here, we will limit ourselves to just one sample of reducing the Minister's problem to the  $k$ \_Clique problem (Figure 16) [13,14]. It can also use the queen

problem in finding the largest complete subgraph of this graph, which is an  $np$ -complete problem.

Applying verification and deducting the index values of the problem leads to the output (+1, +2, -2, -1) according to the example mentioned in the quadratic rule and the problem-solving answers which are (2, 1, 4, 3). We can display Xs from

problem 3sat and use it [1, 15]. It means that we reach the value of zero if the problem is correct and also if we display the

output of the previous problem as specific intermediate values  $(\bar{x}_1, \bar{x}_2, x_3, x_4)=0$ .

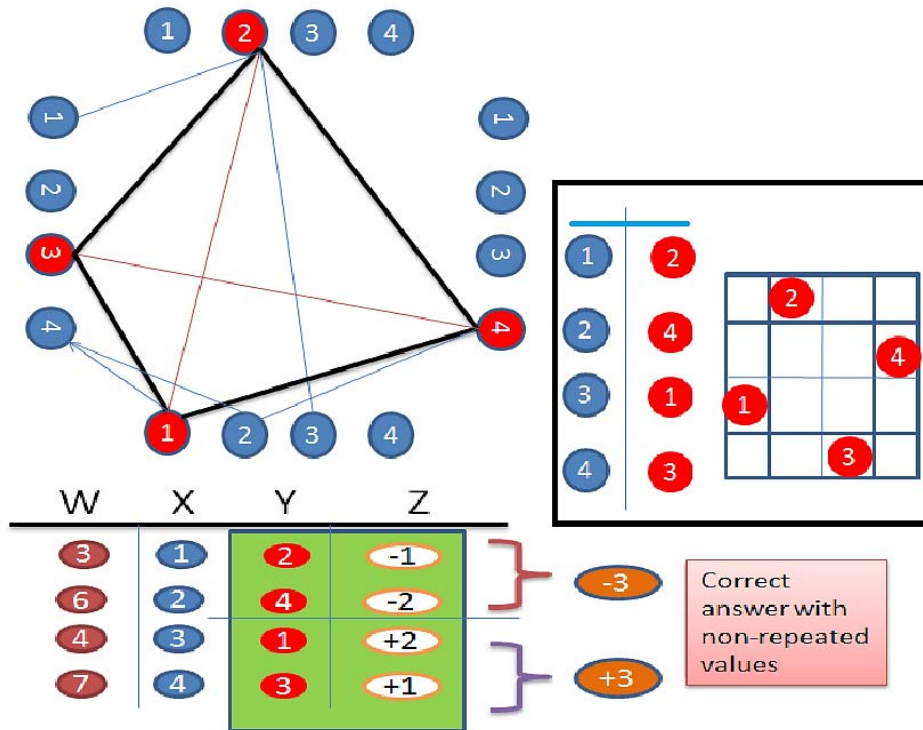


Figure 16 Schematic in reducing n-queens problem to k-cliques problem

### Findings

The use of quadratic progression, capacity estimation, quadratic and fractal relationships [9, 10], rotation and repetition [11], and balancing are among the methods of solving the problem, which can be used in the form of several techniques. Here, the contents will be summarized and presented as a general plan called the predictive method. In this method, the properties of collections (Subtraction, Addition, Communal, and Community) were used in some way (Figure 2). The general description of the problem is as follows:

1. Develop 2 ordered arrays of that chess queen from an  $n \times n$  problem
2. Choosing a number according to priority from the smallest to the largest number from the set of n-queens.
3. Creating a set of candidate squares A on the right side of the problem and a set of critical and unusable B squares on the left side of the problem, so that is in the set on the right side of the problem should not be in the set on the left side of the problem, so that everything that is in the set of the right side of the problem should not be in the set of the left side of the problem [12], i. e., as the difference of sets (Figure 16).
4. In each row of the problem, the selected number of the previous row with a lower value and a higher value is placed

on the side of the critical squares of group B (left side of Figure 16). In this method, in the next rows, with each selection, one value less and one value more than the previous row is placed in the critical point of the set (left side) and it continues until the end.

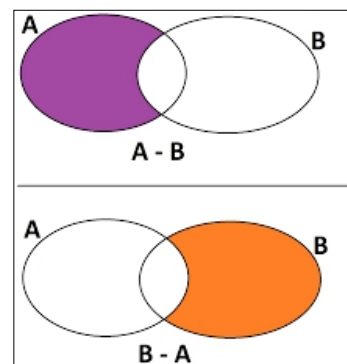


Figure 17: schematic a sample of the difference between two groups

In this case, in the right group in each row, the critical points are removed and the correct values can be selected. It will be possible to predict the critical points according to the values in the set of the left and right sides of the problem. For example,

we are not allowed to select square 6 according to (Figure 3) in rows 5, 6, and 7, because square 6 is mentioned repeatedly in these rows, and it is an element of critical squares, while there is only one selectable value in row 8. It is the common square 6, which is chosen as the correct square. In the following, [13] every time a value from the set of selectable numbers on the right side is reduced and added to the list of critical squares. That is, we will have the difference between the group of critical squares B and the group of candidate squares A.

5. It should always be one or more spaces between the optional numbers. For example, for 2, 4, 8, and 10, otherwise, there is a possibility of coincidence of numbers and transfer to the

critical points list. It is possible to compare the selected numbers with the selected row in the choices only once. For example, we get the value of zero according to the figure of row 2 with the value of 2 as their difference. This means that the selected square is on the main diameter of the problem, which is saved for the next time and should not be repeated.

6. The set of fractions on the right side of the problem in the first half should always be opposite to the set of fractions on the right side of the second half of the problem (Figure 17). In the equation below, the values of  $\bar{x}$ ,  $x$ , which is the set of  $-4+4=0$ , that is, the right side of the problem always ends with the value of zero according to this equation.

Critical squares B		A candidate squares								sums		substractions				
		index	1	2	3	4	5	6	7	8						
		1	7	1	2	3	4	5	6	7	8	8	1	7	-6	
		2	2	1	2	3	4	5	6	7	8	4	2	2	0	$-4=-6+0-1+3$
		3	4	1		3	4	5	6	7	8	7	3	4	-1	$\sim x$
		4	1	1		3	5	6	7	8	5	4	1	3	$-4+4=0$	$x + \bar{x} = 1$
		5	8			3	5	6	7	8	13	5	8	-3		
		6	5			3	5	6	7	8	11	6	5	1	$4=-3+1+4+2$	
		7	3			3	5	6	7	8	10	7	3	4	$x$	
		8	6			3	5	6	7	8	14	8	6	2		

Figure 18: How to calculate the predictive method, conversion, and proofing

7. There should not be repeated values on both sides of the problem (subtraction and addition).

8. It is possible to use set command codes (SET) in the design of the problem-solving algorithm. In this pattern,  $n=8$  is assumed by default  $A = [1,2,3,4,5,6,7,8]$  as candidate squares A and  $B = [1,2,3,4,5,6,7,8]$  as the critical squares B, which at the beginning of the problem  $B = \text{null}$  is based on the counting numbers of bits due to properties such as non-repetition of the members of the sets and their small size and speed. It has the capabilities of association, sharing, differentiation, commonality, contradiction, and bit search, which can be a great choice for problem-solving.

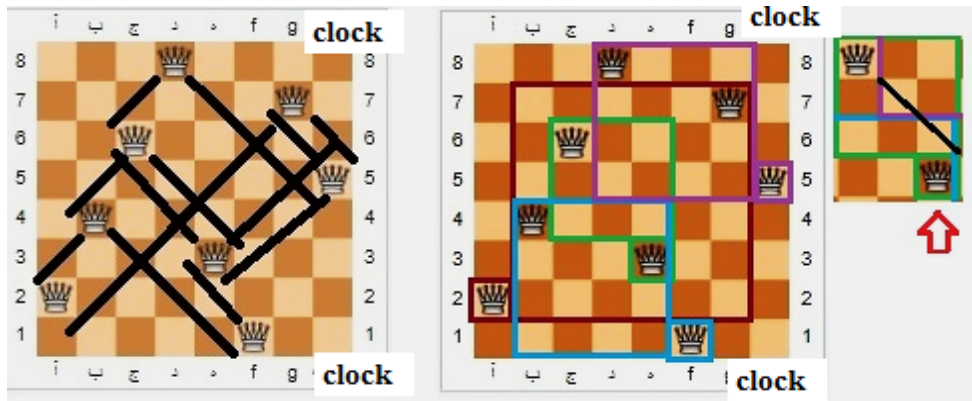
9. The possibility of problem reduction [14] to other  $np\_complete$  problems considering the set fraction set on the right side of the problem according to the example is  $C_i = \bar{x}_1, x_2, \bar{x}_3, x_4, \bar{x}_5, x_6, x_7, x_8$ , which can be used in sat3 calculations according to the laws of Boolean algebra and Cook's theorem. In other words,  $C_1 = \bar{x}_1, x_2, \bar{x}_3, x_4$  in the upper half and  $C_2 = \bar{x}_5, x_6, x_7, x_8$  in the lower half can be 2, c 1c at the opposite point.

In the above example (Figure 17), the rules of the previous chapter and the possibility of using them simultaneously and step by step are used to get the correct answer for the problem. Rules such as capacity estimation, which are mostly used in the second half of the table (bottom), are the critical part of the

problem in finding the correct answers. The values that are candidates for selection in a row will increase the problem sensitivity based on the exploration on the right side of the table because the *inaccurate* estimation will deviate from the problem-solving path (Figure 17).

Square 8 in lines 5 and 6, square 5 in lines 6 and 7, and squares 3 and 6 in lines 7 and square 8 are not critical points. The pair of rows 6 and 5 can be encountered by a square with the value 8, which is independently considered as a choice, where each independent square (8, 5, 6, 3) with a given pair of rows can estimate a correct answer.

11. We can get an attractive and always correct relationship by combining these rules if we refer to the limit rule and the rule between the problem diagonals and the rule of square progression that was mentioned in the previous chapter. We will see an oblique asymptotic relationship in the square diameter of the origin and destination chess piece (obliquely) with this combination in solving the problem on the right side (Figure 18) to the oblique asymptote of the destination chess piece compared to the square diameter created by the origin chess piece, which refers to the equation of  $n^2$ . It can be said that it establishes a regular and always correct relationship between all candidate chess pieces (for example, in the 6-queen problem).

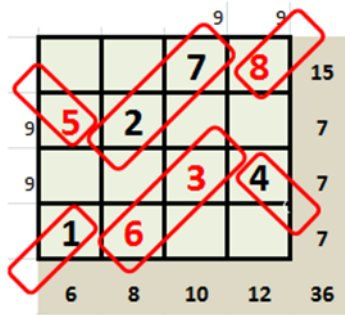


**Figure 19: An example of the combination of the interval rule and the rule between the diagonals of the problem and the square progression rule, which lead to the oblique asymptote of the destination chess piece compared to the diameter of the square created by the source chess piece, which refers to then  $n^2$  relationship**

12. We see a concordance and balance in the sums in the columns and rows and even the diagonals of the table using quadratic rules (Figure 18). Of course, it is necessary to act in the same way as Kano's table in Boolean algebra, so that we finally reach the value of  $n \times n$  using the accumulation of values in diagonals. In quadratic rules, the appropriate answer was obtained using half of the correct answers in solving the

problem randomly or sequentially by moving along the columns of the problem. In the following, the final answer was obtained with a Fact checking.

The statistical analysis and mathematical analysis and algorithm of the problem will be different according to the rule and the relations of applying the problem. The problem complexity will be debatable due to these changes in the design of the algorithm.



**Figure 20: A model of the quartile rule according to the conditions used in the form of Carnot's table and Boolean algebra, which can be used in addition to using the product of row and column sums, the product of diagonal sums ( $2 \times 2$ ).**

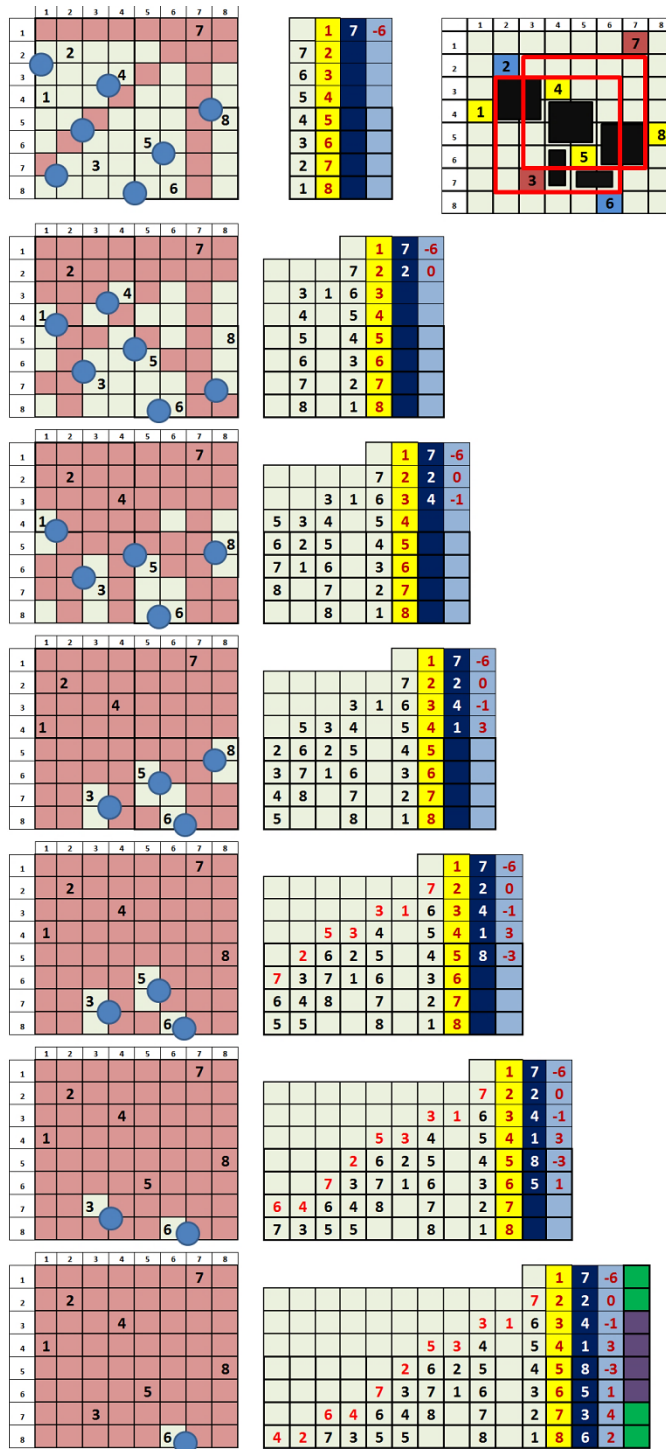


Figure 21. An overview of the rules governing the problem, such as the capacity estimation rule, the quadratic rule, the quadratic progression rule, etc., is presented in the form of an 8x8 table that can be used if necessary.

**Note:** The proposed algorithm is a predictive method using one of the techniques and rules mentioned in the text.

Var

A\_Set , index\_set , B\_Set = {1,2,3,...n};

function checknode (v)

if (there is a solution A\_Set(i)) then

add index\_set(i) and add B\_set(i) and delete A\_set(i) ;

write the solution index\_set(i) ;

else

for (each child u of v)

checknode(u);

main(n)

var i, j=n

For i=(1 to n)

```

For j= (1 to i)
  add B_set(i) = index_set(i) - index_set(j) ;
  checknode (v);
Select the of suggested techniques;
End;

```

### Conclusion

In the present research, the recognition and acquisition of some of the rules and relationships governing the n-queen problem will be discussed, considering the lack of information sources in sequential solving the n-queen problem. A description and analysis and a suitable rule are provided with reviews for each of them. It should be noted that every time we use the ruling rules in this problem, whether tangible or intangible, we often witness balance in the relations between the chess pieces so that these rules do not violate or reject each other. The equation of  $n^2$ , which plays a fundamental role in advancing the solution of this problem with the rules of square progression

[15 and 16], finally limits the calculation space to  $(4^n)/2$ , even with the help of the mentioned rules in the best conditions of  $n^4$  reached acceptable results.

We can use the subtraction result of the squares, which leads to the number 2 to identify and select the candidate squares, solve the problem with the rule between diagonals, solving the problem with the return relationship  $(4^n)/2$ , image processing with  $n^2$  or  $\sqrt{n}$ , to use a predefined database and in the worst case discover safer h squares and replace them.

The current techniques of each square in each row have only 4 selection modes, and as n becomes larger, this rule is preserved and does not grow exponentially [17]. The value of 2, can play an important role in the rules of balance and quadratic and square progression, the minimum distance between candidate numbers that exceed the value of one, and the effect of decreasing and increasing the calculation with -1 and +1. Finally, these rules confirm the other problems presented in this research.

**Table 2: How to calculate the previous methods and the growth process and the general rule between them**

The number of promising nodes found by the backward algorithm	The number of nodes checked by the backward algorithm	The number of candidate solutions checked by the algorithm *2	The number of nodes checked by the algorithm *1	n
17	61	24	341	4
2057	15721	40,320	19,173,961	8
$8.56 \times 10^5$	$1.01 \times 10^7$	$4.79 \times 10^8$	$9.73 \times 10^{12}$	12
$2.74 \times 10^7$	$3.78 \times 10^8$	$8.72 \times 10^{10}$	$1.20 \times 10^{16}$	14
* The values indicate the number of checks required to find all solutions				
* Algorithm 1 shows the depth search in the solution space tree				
* Algorithm n! 2 develops a candidate solution that places each chess queen in a different row and column.				

There is no doubt that there is a definite solution for the  $p=np$  problem and which topic and rule can perform better in solving the problem faster. It has a problem depending on the type of use and statistical and experimental analysis and discovery of the ruling order, and in most cases, the use of mathematical relationships can be used as a general rule. Statistical analysis is used to achieve the percentage reduction of the complexity of an algorithm.

In this research, it is possible to address the ultimate goal of the equality of  $p=np$  and finally its great impact on humanity by changing the angle and attitude in solving the problem using diverse and curious creativity and the mentioned findings and topics.

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